**Beginning Algebra Made Useful**

**Chapter 3**

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# Chapter 3 Using Linear Functions to Solve Problems

## 3.1 Linear Equations Arising in Practical Situations

In Chapter 2, we became familiar with graphs, tables, and equations, particularly where each represents a linear function. We also investigated many contexts that can be modeled by linear functions. In this chapter, we introduce several others. Our goals for Chapter 3 include solving equations and interpreting the solutions to help you make decisions about each context. The examples we study might trigger your recall of situations in your life that can be modeled by linear functions. Let’s begin with pricing pizza!

### Activity: Pricing Pizzas

Pizza restaurants let you customize your pizza. You can also order specialty pizzas. Are you getting a good deal? If so, how good is the deal are you getting? If not, why is the deal not so good?

Work together to analyze the pricing of a pizza restaurant. Use a pizza restaurant’s website for a location close to you as sometimes different locations of the same chain charge different prices. Each group should choose a different pizza size or crust type to investigate. Fill in the table as you go and share your work with the class.

1. Play with a pizza restaurant’s online menu to determine prices requested in the table below. Check prices on more than one ingredient to determine if the prices are the same or different based on ingredient type. For example, are meats more expensive than veggies? Fill-in the table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Description** | **Small,  original crust** | **Medium, original crust** | **Large,  original crust** | **Large,  thin crust** | **Extra Large, original crust** |
| Diameter of pizza | 10 inches | 12 inches | 14 inches | 14 inches | 16 inches |
| Price of Plain Cheese Pizza |  |  |  |  |  |
| Cost of one topping on pizza |  |  |  |  |  |
|  |  |  |  |  |  |
| Price of Cheese pizza with  1 topping |  |  |  |  |  |
| Price of Cheese pizza with  2 toppings |  |  |  |  |  |
| Price of Cheese pizza with  3 toppings |  |  |  |  |  |
| Price of Cheese pizza with  4 toppings |  |  |  |  |  |
| Price of Cheese pizza with  5 toppings |  |  |  |  |  |
| Price, *P*, of Cheese pizza with *t* toppings (equation) |  |  |  |  |  |

2. For each type of pizza, find an equation that gives the price of a pizza with *t* toppings. Write the equation in the last row of the table above.

3. Compare the equations for each pizza type. If you graph these equations, which graph would you expect to be steepest? Why?

4. Using an electronic graphing tool and different colors for each pizza size/crust type, graph all of the pizza data. Choose appropriate scales for each axis. Label the scales and titles. Label each graph with pizza size and crust type. Compare the graphs. What do you notice?

5. a. Which graph is the steepest? What is the slope of the steepest graph?

b. How does the slope show up in the table?

c. How does the slope show up in the equation?

6. a. Which graph has the largest *y*-intercept?

b. How can you tell from the graph?

c. How can you tell from the table?

d. How can you tell from the equation?

7. Choose two different specialty pizzas from the same pizza restaurant as previous problems. For each specialty pizza:

a. Use the pizza restaurant’s website to find the price for each pizza size. Fill in the table.

b. Determine the price if you customized the pizza instead of ordering the specialty pizza.

c. Which is the better deal? Why?

8. Choose on size of pizza. Show your work as you answer each problem.

a. Determine the price of a pizza with 8 toppings.

b. Determine the number of toppings you can get for $25.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Description** | **Small,  original crust** | **Medium, original crust** | **Large,  original crust** | **Large,  thin crust** | **Extra Large, original crust** |
| Price of specialty pizza (list name): |  |  |  |  |  |
| Number of toppings on pizza |  |  |  |  |  |
| Customized price |  |  |  |  |  |
| Which is the better deal? Why? |  |  |  |  |  |
|  |  |  |  |  |  |
| Price of specialty pizza (list): |  |  |  |  |  |
| Number of toppings on pizza |  |  |  |  |  |
| Customized price |  |  |  |  |  |
| Which is the better deal? |  |  |  |  |  |
| Price of a pizza with 8 toppings |  |  |  |  |  |
| Number of toppings you can get for $25 |  |  |  |  |  |

Share how you determined your responses in the second table, and particularly for the solutions to problem 8. Keep track of approaches different students used to solve problems 8 and 9. What similarities were there in solution strategies?

What strategy do you want to remember when determining the price of a pizza with 8 toppings? What strategy do you want to remember when determining how many toppings you can afford for $25? Apply those strategies to solve the following problems:

* You’re going to the state fair. The entrance fee is $15. Each ride costs $3.
  + Determine an equation that gives the total amount you will pay for entrance and rides based on the number of rides.
  + Use the equation to determine the amount you will pay for admission and rides if you ride 6 rides.
  + You have $45 to spend for the fair. How many rides can you take?
* Your favorite clothing store has a storewide 20% off sale going on.
  + Determine an equation that works for the sale for every original price of items.
  + What is the sale price for a $35 shirt?
  + What was the original price of an item you paid $47 to buy?

How does the context help you decide how to solve each problem? How does the context help you determine for which variable you’re solving?

### 3.1 Homework

1. A local grocery store sells coffee by the cup on an honor system. A customer can choose from a variety of K-cups to make the cup of coffee. The store charges 50¢ per cup.

a. Assuming customers are honest and pay 50¢ for each cup consumed, how much will the store take in based on any number of individual cups of coffee sold?

b. If 105 people buy single cups of coffee, how much will the store take in?

c. How many cups must the store sell in order to take in $250?

d. The store also sells K-cups in cartons for customers to take home to make their own coffee. One carton of a popular brand contains 18 K-cups and costs $8.59. What is the per cup cost for a customer who buys this product?

e. Determine an equation that shows the cost per cup over time if customers buy the product in problem 1d to make coffee at home.

f. Use the equation you found in problem 1e to determine how many cups of coffee a customer can make at home for $250. How many cartons of coffee is this?

2. a. Complete the student page, *Solving Equations Hands-On Minds-On*. Use an algebra tile app online to model equations with integer coefficients. For example:

<http://media.mivu.org/mvu_pd/a4a/homework/index.html>

<http://www.mathplayground.com/AlgebraEquations.html>

b. If you have trouble solving any of the problems on the student page, use one of the apps to try additional problems until you are comfortable solving them.

c. Explain how to solve a linear equation.

3. Solve each equation. Show your steps one at a time. Put your solution back into the original equation and show that you are correct. If you are not correct, illustrate your work using pawns for *x* and cubes for constants. Use the illustration to solve the equation then revisit your algebraic work, find your error, and try again. Think about a context such as balancing a scale to help you. Be ready to convince others that your work is correct.

a. 4*x* + 3 = 7*x*  b. 3*x* + 2 = 5*x* + 1 c. –4*x* + 3 = –*x* + 9

d. 3*x* + 2 = 2(*x* + 1) e. 2(*x* + 6) = 5(*x* – 3) f. 5(7 + 4*x*) = 5(3*x* + 10)

g. –2(*x* – 3) = –(*x* + 7) h. 8 + 2*x* = 1.4*x* + 11 i. 200 – 7.5*x* = 35

j. 4(*x* – 3) + 2 = 2(3*x* + 5)

k. What questions do you have regarding solving equations? Share your work with your group. Resolve any differences.

Solving Equations – Hands and Minds On

When set in context, it is easier to decide how to solve an equation. For example, when trying to determine how many toppings you can get on a large original pizza for $25, you might have thought about the solution in one of the following ways:

1. An original crust large cheese pizza costs $10.99. To figure out the number of toppings I can get, I see that I have $25 – $10.99 = $14.01 to spend on toppings. Each topping costs $1.50. (Parts a, b, and c are different possible ways to proceed from here.)

a. I can find the number of toppings I can afford by dividing $14.01 by $1.50. This gives me 9.34. The pizza restaurant will only allow me to buy whole numbers of toppings so I can get 9 toppings on a $25 large original crust pizza.

b. If I start with $14.01 and subtract 1.50 until I can’t subtract it anymore, I can do that 9 times, so I can afford 9 toppings on my pizza if I have $25 to spend.

c. Two toppings cost $3. $3 x 4 = $12 with $2.01 left over for one more topping. So I can afford 8 toppings + 1 topping = 9 toppings for $25.

2. The equation for the large pizza is *P* = 10.99 + 1.50*t*. The price is $25; this is *P. T*he equation becomes 25 = 10.99 + 1.50*t*. Subtract 10.99 from each side to get 14.01 = 1.50*t*. Now divide both sides by 1.50 to get *t* = 9.34. This problem is about numbers of toppings on pizzas. The pizza restaurant makes you pay for full toppings, so *t* = 9.

Notice that it helps to keep the context in mind. The context helps you make sense of the operations you are using. This allows you to be very flexible with the ways you solve the problem because you know what each of the parts represent.

Here is another context for solving equations: Let a pawn represent an unknown value of *x*. Let small cubes represent 1. Think about each side of an equation as sitting on a balance scale. Because one side equals the other side of the equation, the scale is balanced. In order to keep a scale balanced, it is necessary to add the same thing to both sides, remove the same thing from both sides, or divide both sides into the same number of groups then remove parts that equal each other from both sides, leaving only one group on each side of the scale. Following are examples that we will solve using this context. (The problems with negative signs and the problems with subtraction symbols require 2 different colors of cubes (pawns), one color to represent positive one (+*x*), and one color to represent negative one (–*x)*.)

|  |  |
| --- | --- |
| *x* + 1 = 5 | 2*x* = 6 |
| 2*x* + 1 = 7 | 2*x* + 1 = 3*x* |
| 2*x* + 7 = 3*x* | 2*x* + 7 = 3*x +* 5 |
| 2(*x* + 2) = 3*x* + 2 | 1 + –1 = *x* + –*x* |
| *x* – 1 = 5 | 2*x* – 1 = 3 |
| 3(*x* – 1) = *x* + 3 | –2*x* + 6 = *x* |

Now resolve the above problems, this time thinking of *x* as some unknown number of bananas. The constants (those numbers not multiplied by *x*) are known numbers of bananas. (Omit the problems with subtraction symbols and the problems with negative signs.)

## 3.2 Solving Linear Equations: Packaging Stacked Cups

As you have experienced, there are many examples from your life that are reasonably modeled by linear functions. In this lesson, you will consider a packaging problem. As you solve the problem, think of other items of similar size that you stack and store.

### Activity: Packaging Stacked Cups

Using a stack of at least 15 same-size cups, gather the data requested in problem 1. Share the data with the class. Find the average data for the groups that measured the same types of cups. Use the average data to complete problems 2 through 7.

Consider the second size of cups. Answer problem 8 for these cups in comparison with the smaller cups.

What other questions can you ask and answer?

Packaging Stacked Cups

A company manufactures disposable cups and rectangular cardboard cartons to package them. The cups come in several sizes. You job is to develop a mathematical model that will help determine the relationship between the inside height, *h*, of the carton and the number of cups, *c*, it will hold if the cups are to be stacked in the carton.

1. You have been provided two different types of disposable cups. For the small cup, complete the table showing the relationship between the number of cups, *c*, in a stack and the total height in centimeters, *h*, of the stack. Show accuracy to the nearest tenth of a centimeter.

|  |  |
| --- | --- |
| **Number of Cups, *c*** | **Height of Stack, *h*, in cm** |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| *c* |  |

2. Graph the data. Label the axes showing the variables and scales.

3. Find a formula that allows you to predict the height of the stack of cups based on the number of cups.

4. Predict the height of a stack of:

a. 10 cups b. 26 cups

5. What physical features of a cup are relevant to how high the cups stack? In terms of the physical features of a cup, write a general rule for the height of a stack based on these physical features.

6. Someone said, “If you double the number of cups in a stack, the height of the stack doubles.” Is this thinking right or wrong? Why do you think so?

7. Using the graph in problem 2, how many cups will cartons of the following heights hold?

a. 30 cm b. 50 cm

8. Use the large cups.

a. How would the graph be different from the graph of the small cups in terms of steepness? Explain.

b. Sketch an approximate graph for the large cups on the same set of axes as the graph in problem 2.

### 3.2 Homework

1. Graph *Packaging Stacked Cups* average class data and your group data.

a. On Desmos, type the class data into a table, then type, *y* = *mx* + *b*. Choose ‘both’ to set up sliders for *m* and *b*. Then play with both *m* and *b* until you find a line that seems to best fit the data. Record the equation for the line.

b. Repeat problem 1a for your group’s data. Some equations exactly fit the data. If this is the case for your group, also find the equation algebraically.

c. Compare the equations you found for the average class data and for your group data. Which one do you think is more accurate? Why?

d. Save your work on this problem. It will be revisited in a later lesson.

2. Data for *Packaging Stacked Cups* from a previous class is listed below. The table shows the relationship between the number of cups, *c*, in a stack and the total height in centimeters, *h*, of the stack with accuracy to the nearest tenth of a centimeter.

Previous Class Data for *Packaging Stacked Cups*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Number of Cups, *c*** | **Height of Stack, *h*, in cm** | | | | | | |
| **Group 1** | **Group 2** | **Group 3** | **Group 4** | **Group 5** | **Group 6** | **Average** |
| 1 | 7 | 7 | 7.2 | 7 | 7.4 | 7 |  |
| 2 | 7.5 | 7.5 | 7.6 | 7.5 | 7.9 | 7.5 |  |
| 3 | 8 | 8 | 8.1 | 8 | 8.1 | 8 |  |
| 4 | 8.5 | 8.5 | 8.6 | 8.5 | 8.6 | 8.5 |  |
| 5 | 9 | 9 | 9 | 9 | 9.1 | 9.5 |  |
| 6 | 9.5 | 9.5 | 9.5 | 9.5 | 9.7 | 9.5 |  |
| *c* |  |  |  |  |  |  |  |

a. In the table above, which set of data is perfectly linear? How do you know?

b. Find an equation to fit the linear data you chose in problem 2a (see problem 1a). Explain both the slope and *y*-intercept in terms of stacked cups.

c. Use the equation you found to determine the internal height of a package that will contain 50 cups. Show and explain your work.

d. Use the equation you found to determine the number of stacked cups that will fit inside a package with internal height of 100 cm. Show and explain your work.

e. Find the average class data for the data table in problem 2. Fit an equation to the data (see problem 1a).

f. Solve problems 2c and 2d using the equation in problem 2e.

3. You pay a $5 entrance fee and $3 per ride at a fair.

a. Find an equation that fits this situation. How do you know your equation is correct?

b. You want to go on 7 rides at the fair (see problem 1a). How much money do you need to bring along?

c. You go to the fair with $15 in your pocket. How many rides can you ride?

d. Solve the equation in problem 3a for the independent variable. What does this new equation mean?

4. Solve each of the following equations for *x,* the independent variable.

a. *y* = 3*x* – 4 b. *y* = c. *y* = –4.2*x* + 5

## 3.3 Eyeballing Lines of Best Fit

Sometimes data is not perfectly linear, but has a linear trend. An example follows.

### Activity: Pricing Canisters

The height and price of each of the canisters in the picture is provided in the table:

|  |  |
| --- | --- |
| **Height (in inches)** | **Price (in dollars)** |
| 7 | 24.95 |
| 8.75 | 29.95 |
| 12 | 39.95 |

* Would you expect the data in the table to be linear? Why or why not?
* Is the relationship between the height and the price of the canisters linear? Why or why not?
* Is the data perfectly linear? If not, which canister seems to be priced too high or too low? Explain your thinking.

In this lesson, we will consider other experiments and data sets. We will first determine if we think a linear trend would be a good fit. If so, we will eyeball a line of best fit, determine an equation for the eyeballed line, and use the line to test data and make predictions.

A line of best fit has these properties:

1. The direction of the line is evident from the data.

2. As many points as possible should be on the line as long as the same number of points are above and below it.

3. The points above the line should not be concentrated at one end of the line nor should the points below the line.

### Activity: Gulliver Graphs

In *Gulliver’s Travels*, the Lilliputians made a coat for Gulliver that fit him perfectly. They did this after taking only one measurement, the circumference of Gulliver’s thumb. The Lilliputians then used the following well-known Lilliputian “Rule of Thumb”:

The circumferences of each body part are related as in the following equations:

Wrist = 2 • Thumb, Neck = 2 • Wrist, and Waist = 2 • Neck

1. By gender, gather thumb, wrist, and neck measurements, in centimeters, for at least 20 adults. Measure the thumb between the knuckles. Measure the wrist on the arm just above the wrist bones. Measure the neck just below the chin.

2. Graph the data for (Thumb, Wrist), one graph each for females, males, and entire data set. Use a different color for each data set.

3. Draw lines that best fit each data set. Explain why you think each line is a reasonable fit.

4. Test the Lilliputian “Rule of Thumb” for:

a. The female sample,

b. The male sample,

c. The entire sample.

5. Repeat problems 2 through 4 for the data, (Wrist, Neck) and for the data, (Thumb, Neck).

6. Were the Lilliputians lucky that Gulliver’s coat fit or were they quite clever? Explain.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Female** | | | **Male** | | |
| **Thumb** | **Wrist** | **Neck** | **Thumb** | **Wrist** | **Neck** |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
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Functions for Females: Functions for Males: Functions for Entire Sample:

W(T) = W(T) = W(T) =

N(W) = N(W) = N(W) =

N(T) = N(T) = N(T) =

### A Word about Function Notation

When you were asked to record the functions you found for the *Gulliver Graphs*, did you find the notation a bit strange? Notice that the notation gives important information about the columns of data being related to each other. W(T) indicates that we want to know the wrist measurement in terms of the thumb measurement, instead of just using W. We read the symbols, W of T. What do you think N(W) represents? N(T)?

If we want to be very specific, we could indicate WF(TF) to indicate the that we want to know the wrist measurements for females in terms of the thumb measurements for females. How would we write the wrist measurement for males in terms of the thumb measurements for males? We are not using subscript notation in *Gulliver Graphs*, instead opting to use headings to make clear which data sets are being related.

From this point on, we will use function notation to avoid ambiguity in cases where it is necessary to make clear which quantities we are relating.

### 3.3 Homework

1. The table shows U.S. shoe sizes for women as compared with foot lengths in inches and centimeters and Euro and UK shoe sizes (<http://www.zappos.com/c/shoe-size-conversion>, retrieved April 22, 2020). Let U.S. shoe sizes be the independent variable. Choose a variable to represent each column of data. Use function notation when recording equations.

a. Graph the data, (US sizes, length measurement) for inches or centimeters. Is the data linear? Why or why not? Estimate a line of best fit. Find an equation for the line.

b. Graph the data, (US sizes, Other sizes) for Euro or UK. If you choose Euro Sizes, what should you do to accommodate the range of sizes shown for US half sizes? Is the data linear? Why or why not? Estimate a line of best fit. Find an equation for the line.

c. Are any of the data sets perfectly linear? If so, which ones? If not, what do you think causes the data set to not be perfectly linear?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **US Sizes** | **Inches** | **CM** | **Euro Sizes** | **UK Sizes** |
| **4** | 8.1875 | 20.8 | 35 | 2 |
| **4.5** | 8.375 | 21.3 | 35 | 2.5 |
| **5** | 8.5 | 21.6 | 35-36 | 3 |
| **5.5** | 8.75 | 22.2 | 36 | 3.5 |
| **6** | 8.875 | 22.5 | 36-37 | 4 |
| **6.5** | 9.0625 | 23 | 37 | 4.5 |
| **7** | 9.25 | 23.5 | 37-38 | 5 |
| **7.5** | 9.375 | 23.8 | 38 | 5.5 |
| **8** | 9.5 | 24.1 | 38-39 | 6 |
| **8.5** | 9.6875 | 24.6 | 39 | 6.5 |
| **9** | 9.875 | 25.1 | 39-40 | 7 |
| **9.5** | 10 | 25.4 | 40 | 7.5 |
| **10** | 10.1875 | 25.9 | 40-41 | 8 |
| **10.5** | 10.3125 | 26.2 | 41 | 8.5 |
| **11** | 10.5 | 26.7 | 41-42 | 9 |
| **11.5** | 10.6875 | 27.1 | 42 | 9.5 |
| **12** | 10.875 | 27.6 | 42-43 | 10 |

2. Have you played Monopoly? The graph shows the number of the space (after GO) and the price for the space on a Monopoly game board. GO is space 0. (You can look up the gameboard on the Internet.)

a. Does a linear model appear to fit the data? Explain your thinking.

b. On the graph provided, draw your best guess for a line of best fit for the data (see p. 118 for a reminder for properties of a line of best fit).

c. Find an equation for your hand-drawn line. Show your work!

d. What does the slope mean in terms of Monopoly space numbers and prices?

e. What does the *y*-intercept mean in terms of Monopoly space numbers and prices?

f. What is the independent variable? Why do you think so?

3. The *domain* of a function is the set of all possible values of the independent variable (*x* or whatever variable you are using for the context) for which the function is defined. The *range* is the set of possible values of the dependent variable (usually *y*, sometimes expressed as *f*(*x*)) that result from using the same function.

a. What is the domain of the function in the graph in problem 2 in terms of the context?

b. What is the range of the function in the graph in problem 2 in terms of the context?

## 3.4 Linear Regression and Lines of Best Fit

In the activities and homework for Lesson 3.3, you might have found a different equation than other members of your group or class. Even if your equations had similar slopes or *y*-intercepts, it might be disconcerting to use different equations to model the same set of data.

Statisticians worked on the problem of finding lines of best fit over many years. Initially, the idea was to get a line that had as many points above the line as below so that the sum of the distances of points away from the line added to 0. Even though the process worked fairly well, the same problem occurred as in our work with eyeballed lines of best fit. Scientists were not satisfied with estimates that worked most of the time. They wanted precision and a method that was mathematically justifiable. Carl Friedrich Gauss developed the method of least squares to solve celestial problems.

To understand the method of least squares, consider an example. In Figure 1, three points and an eyeballed line of best fit are indicated. Each point is connected to the line with a segment the length of which is a residual, the directed distance of the point from the eyeballed line of best fit; a residual is positive if the point lies above the line and negative if the point lies below the line. Notice that the sum of the residuals is 0. In Figure 2, each residual is the same as in Figure 1 but this time, a square with side length equal to the residual is drawn. In the method of least squares, the squares of the residuals are found; these are the areas of the squares. The one line that best fits all of the points is the line for which the sum of the areas of the squares is least.



Figure 1 Figure 2

### Activity: *The Wave*

|  |  |
| --- | --- |
| **Number of Participants** | **Time to Complete the Wave** |
| 5 |  |
| 10 |  |
| 15 |  |
| 20 |  |
| 25 |  |
| 30 |  |
| 35 |  |

Let’s try another experiment. Think about the Wave as it happens at a football game. Someone starts it and gets most of the people in the stadium standing up and sitting down as the wave approaches then leaves them. We’ll practice the wave in the same way, keeping track of how long it takes to complete the wave based on the number of participants in the wave.

Use the table at right to record the amount of time it takes to complete a wave for the given number of participants.

* Plot the data.
* Describe the graph.
* Eyeball a line of best fit.
* Find the equation of the line you found.
* What does the slope represent in this context?
* What does the *y*-intercept represent in this context?

### Activity: Regression with Desmos and Graphing Calculators

As you might have guessed, there are electronic tools to find regression equations and lines of best fit. Both Desmos and graphing calculators have tools that will determine lines of best fit using the method of least squares. The graphing calculator directions are included at the end of the homework in this lesson. Desmos includes directions in the on-line graphing calculator. To find the Regression directions on Desmos, click on the ? button in the upper right corner of the screen. At the top of the screen that appears, there is a list of Tours. Choose *Regressions*. Follow the directions using the data set Desmos provides. Use Desmos’ directions to enter and analyze additional data sets.

In this lesson, use Desmos or a graphing calculator to find a regression equation to fit the Wave data. How close does your eyeballed line of best fit approximate the regression line? How long would it take to complete the wave if 100 people participated? How many people would have to participate to keep the wave going for 10 minutes?

### 3.4 Homework

1. Each space in the Monopoly board game is numbered with GO being space 0. The price of the property by space number is provided.

a. Plot the data.

b. Use the regression capability of an electronic tool to find a regression equation to fit the data at right. Record the equation.

c. What is the slope? What does the slope represent in this context?

d. What is the *y*-intercept? What does the   
*y*-intercept represent in this context?

e. Compare the slope and *y-*intercept from your regression line with those of the eyeballed line of best fit (see 3.3 Homework, problem 2). How close was your eyeballed line to the line you found electronically? Would your eyeballed line be a reasonable approximation for the regression line? Why or why not?

f. Does the regression line agree with the price for property 18 in the table?

g. What property number would cost $500 if the board was expanded to include more spaces?

|  |  |
| --- | --- |
| **Space  Number** | **Price of  Property** |
| 9 | 120 |
| 14 | 160 |
| 19 | 200 |
| 24 | 240 |
| 29 | 280 |
| 34 | 320 |

2. In the table at right, several *Monopoly* properties were deleted from the table in problem 1. Only the third property in each group of three of the same colored properties remain in this table. Space numbers and prices are provided.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Space  Number** | **Price of  Property** | **Space  Number** | **Price of  Property** |
|  | 1 | 60 | 21 | 220 |
|  | 3 | 60 | 23 | 220 |
|  | 6 | 100 | 24 | 240 |
|  | 8 | 100 | 26 | 260 |
|  | 9 | 120 | 27 | 260 |
|  | 11 | 140 | 29 | 280 |
|  | 13 | 140 | 31 | 300 |
|  | 14 | 160 | 32 | 300 |
|  | 16 | 180 | 34 | 320 |
|  | 18 | 180 | 37 | 350 |
|  | 19 | 200 | 39 | 400 |

a. What is the slope? What does the slope mean in this context?

b. What is the *y*-intercept? What does the *y*-intercept mean in this context?

c. Determine an equation to fit this new table.

d. How does the equation fit the data in this table? Explain.

e. Draw the graph of this line using another color on the scatterplot you created in problem 1. What do you notice?

f. If every space had a price associated with it, how much would space 25 cost?

g. Which space number would cost $500?

3. Revisit the problems below.

* Find regression equations to fit each data set.
* Compare the regression equation with the equation you found earlier. Comment on the accuracy of your previous work when compared with the regression equation.
* Write and answer a meaningful question in context that requires you to solve the regression equation using your choice of a value of the dependent variable.
* Write and answer a meaningful question in context that requires you to solve the regression equation using your choice of a value of the independent variable.

a. *Packaging Stacked Cups* (problems 1 and 2 of 3.2 Homework, pp. 118–119): Choose one of the class data sets. Find a regression equation to fit the average height of the cups depending on the number of cups stacked.

b. *Gulliver Graphs* (Lesson 3.3, pp. 121–122): Choose one of the data sets, (Thumb, Wrist), (Wrist, Neck), or (Thumb, Neck). Find three regression equations, one for each group, female, male, and entire group.

c. U.S. Shoe Sizes (3.3 Homework, problem1, p. 123): Online, find a Men’s Shoe Conversion Chart. Compare men’s shoe sizes to women’s shoe sizes for the same foot lengths. Use foot length in either inches or centimeters as the independent variable and shoe sizes as the dependent variable. Provide a printed list of the data.

4. World records for human accomplishments in sports over time provide interesting data to analyze. Locate a sport of interest for which world record progressions can be found. For example, the men’s world records for the mile run are available online for 1865 to 1999.

a. Provide a printed list of the data.

b. Using an electronic tool, plot the data with year being the independent variable and world record time being the dependent variable.

c. Find a regression equation to fit the data. Graph the equation on the same axes as the scatterplot of the data.

d. Interpret the slope and the *y*-intercept in terms of the years and world record times.

e. What is a reasonable domain for the sport you are investigating? Why do you think so?

f. What is the range corresponding to your chosen domain?

g. Write and answer a meaningful question in context that requires you to solve the regression equation using your choice of a value of the independent variable.

h. Write and answer a meaningful question in context that requires you to solve the regression equation using your choice of a value of the dependent variable.

i. Write a paragraph discussing your findings and including your opinion about whether or not a linear regression equation will be a good predictor for future world record winning times and years.

Entering and Graphing Data and Finding Regression Equations

##### To Enter the Data:

* Press STAT then choose 1: Edit...
* Enter the values for the independent variable (domain) into L1.
* Enter the values for the dependent variable (range) into L2.

##### To Graph the Data:

* Press STAT PLOT (2ND Y=) then choose 1: Plot1... Press ENTER
* Settings:

On,

Scatter plot (first graph type)

Xlist: L1

Ylist: L2

Mark: +

* Press GRAPH

##### To Set the Viewing Window:

* Press WINDOW
* Use the table to determine
* Xmin, Xmax, Xscl
* Ymin, Ymax, Yscl

OR

* Press ZOOM then scroll down to 9: ZoomStat, Press ENTER

##### To Fit a Function to the Data:

* Press STAT
* Move the cursor to the right to highlight CALC
* Scroll down to the function type you want to use, for example, 4: LinReg(ax+b) (to fit a linear function), then Press ENTER.
* Indicate the lists of the data (separated by commas for older TI-84 calculators, L1, L2).
* Choose where you want to place the function equation, for example, Y1. Press VARS, move the cursor to the right to highlight Y-VARS, Press ENTER to choose 1: Function, scroll to highlight the Y-variable you want, then press ENTER again.
* The screen should read:

LinReg(ax+b)

Xlist: L1

Ylist: L2

FreqList:

Store RegEQ: Y1

Calculate

(For older TI-84s, the screen will read 4: LinReg(ax+b) L1, L2, Y1). Press ENTER (to fit a linear function).

* The regression equation will be in Y1 or whatever Y-variable you chose.
* Press GRAPH to plot the regression equation with the data.

## 3.5 Systems of Equations

In Lesson 3.5, we expand our focus. So far in Chapter 3, we have analyzed and solved linear functions and used regression to model linear functions that arise from data. We now consider how two or more linear functions interact with each other. As consumers, we are often faced with choices. In this lesson, we will solve systems of equations in order to determine the best deal for our needs.

Much of the mathematics work we have done this semester has depended on making connections among representations. To study and solve systems of equations, the interrelationships between tables, graphs, and equations are critical. Think about the contexts, and create tables, graphs, and equations to extend what you have learned about linear functions to work with them two or more at a time.

### Activity: Linear Functions and Fitness Plans

By now, you are proficient at finding equations. In the student page, *Linear Functions and Fitness Plans,* you will use tables to locate an interval over which two or more of the fitness plans cost the same amount. You will refine your search for an intersection point with graphs and then by solving pairs of equations.

Linear Functions and Fitness Plans

1. Consider payment plans for three different fitness centers:

* *Shape Up* charges an initiation fee of $39 plus $35 per month per person.
* *Fitness Fun* charges an initiation fee of $75 plus $20 per month per person.
* *Be Your Best* has no initiation fee and charges $52 per month per person.

Complete the table showing the total cost for the first 6 months for each fitness company’s payment plan (initiation fees are only charged once).

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Amount Paid so far (in dollars)** | | |
| **Month** | ***Shape Up*** | ***Fitness Fun*** | ***Be Your Best*** |
| 1 | 74 |  |  |
| 2 | 109 |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| *m* |  |  |  |

2. For each set of data, graph (month, amount paid so far). Label each graph with the company’s name.

3. a. Which company charges the most after 6 months of membership? How do you know?

b. How can you tell from a graph?

c. How can you tell from the table?

4. Which company has the steepest graph? Why does that make sense?

5. What does the *y-*intercept represent for each fitness payment plan?

6. Find an equation that fits each data set. Include the equation in the last row of the table.

7. You have $500 to spend on a fitness plan this year. Show your work.

a. Which fitness plans can you afford?

b. For how many months can you afford each fitness plan?

8. In the summer, you are very active outside. You only want to join a fitness center for 7, 8, or 9 months of the year. The fitness centers offer the same amenities with one exception, *Be Your Best* also has a pool.

a. Determine the cost for all three fitness plans for these numbers of months.

|  |  |  |  |
| --- | --- | --- | --- |
| **Month** | **Amount Paid so far ($)** | | |
| ***Shape Up*** | ***Fitness Fun*** | ***Be Your Best*** |
| 7 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |

b. Which fitness plan would you choose and for how many months? Explain.

9. a. What variable are you solving for in problem 7? How can you tell?

b. What variable are you solving for in problem 8? How can you tell?

### Activity: Systems of Equations in Context: Renting Bicycles

10. a. Solve each equation you found in problem 6 for the variable that represents months.

*Shape Up:*

*Fitness Fun:*

*Be Your Best:*

b. What do these equations allow you to do directly that the equations in problem 6 require more work to do?

11. Consider the following pairs of fitness plans. For what number of months will you pay the same amount for a membership in either plan?

*Shape Up* and *Fitness Fun:*

*Fitness Fun* and *Be Your Best:*

*Be Your Best* and *Shape Up:*

Four different bike rental companies are competing for your business on Mackinac Island. Which one gives the best deal for the number of hours you plan to rent a bike?

This activity helps you determine the best deal. It also helps you make sense of the importance of domain and range both in context and without a context.

When you finish the Renting Bicycles activity, answer the following questions for Lesson 3.5:

* How can you solve a system of two linear equations in two unknowns graphically?
* How can you also solve a system algebraically?

Systems of Equations in Context: Renting Bicycles

You and your friends plan to rent bikes while visiting Mackinac Island.

* Island Rentals charges a base fee of $16 plus $6 per hour per bike.
* Bikes Unlimited does not charge a base fee, but charges $10 per hour per bike.

1. a. You plan to use bikes for 3 hours. Which company is least expensive?

b. Which company should you use if you plan to use bikes for 6 hours?

c. Tegan noticed that if the group uses bikes for exactly 4 hours, the companies charge the same for rentals. Is she correct? Why or why not?

2. a. Find the slope, both intercepts, and meanings of all of these for each company.

b. Find equations for both bike rental companies. Enter them in the table.

c. Graph the equations. Label axes and scales.

d. Plot the data from problem 1 on each graph. Do your equations fit the data you found in problem 1?

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Rental Company** | **Slope** | ***y*-intercept** | ***x*-intercept** | **Equation** |
| Island Rentals |  |  |  |  |
| Bikes Unlimited |  |  |  |  |
| Beckmann’s Bikes |  |  |  |  |
| Colleen’s Cyclery |  |  |  |  |

3. Recall that the *domain* of a function is the set of all possible values of the independent variable (*x* or whatever variable you are using for the context) for which the function is defined. The *range* is the set of possible values of the dependent variable (usually *y*, sometimes expressed as *f*(*x*)) that result from using the same function.

a. Find the domain in terms of the bike rental context.

b. Find the domain disregarding the context.

c. How are these similar? How are they different?

d. Find the range in terms of one of the bike rental companies.

e. Find the range disregarding the context.

f. How are these similar? How are they different?

4. Suppose there are two other bike rental companies on the island.

* Beckmann’s Bikes charges a base fee of $5 and $10 per hour per bike.
* Colleen’s Cyclery charges a base fee of $20 and $5 per hour per bike.

1. How are the charges from these bike rental companies similar to the charges for Island Rentals and Bikes Unlimited?

b. Find equations for each company. Record slopes, intercepts, and equations in the table.

c. Graph the equations on the same coordinate plane you used in problem 2.

d. Compare the graphs of these two companies with those above. What do you notice?

5. a. Are any of the lines in problems 2 or 4 parallel? If so, which ones? How do you know?

b. In the context of bike rentals, what aspects of the bike rental make the lines modeling the rental charges parallel?

6. Write and answer two questions related to bike rentals and the fees the four companies above charge.

### 3.5 Homework

1. Study the lines at right.

a. Estimate the intersection point of the lines.

b. Find at least two grid points on each graph. Do not estimate. Label the points on the graph.

c. Find an equation for each line. This is called a system of equations in two unknowns because the values of *x* and *y* of the intersection point are both unknown.

d. How can you solve the system of equations algebraically?

e. Solve the system algebraically.

f. Compare your answers in problems 1a and 1e. Is your estimate reasonable?

2. Graph the following equations.

*y* = 2*x* – 3

*y* = –0.5*x* + 4

a. What do you know about the intersection point of the two lines?

b. Estimate the intersection point graphically.

c. How does the answer to problem 2a help you determine a way to solve the problem algebraically?

d. Solve the problem algebraically.

e. Replace the variables in both equations with the coordinates of the intersection point. Does the point you found satisfy both equations? Should it? Explain.

3. A video game is available for your cell phone in two versions. The free versions charges $0.99 for each “booster,” an award that helps you win a level more easily. The full version costs $5.99 then charges $0.49 per booster.

a. How much will the game cost you to play overall based on the number of boosters you buy? Fill-in the table. Include equations in the last column for both versions of the game.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Number of Boosters** | **0** | **1** | **2** | **3** | **4** | **5** | ***x*** |
| **Free Version** |  |  |  |  |  |  |  |
| **Full Version** |  |  |  |  |  |  |  |

b. For each of the video games, complete the table with values accurate to at least 2 decimal places. Do not estimate from a graph!

|  |  |  |
| --- | --- | --- |
|  | **Free Version** | **Full Version** |
| ***y-*intercept** |  |  |
| **slope of the line** |  |  |
| **equation that fits the line** |  |  |
| ***x-*intercept** |  |  |

c. Graph the equations. Estimate the coordinates of the intersection point. Write the intersection point as an ordered pair (*x*, *y*).

d. For what number of boosters will both versions of the game cost the same amount? Solve the problem algebraically and say how you know you are correct.

e. What is the domain for the Free Version? Explain.

f. What is the range for the Free Version? Explain.

g. If you ignore the context, what is the domain for the equation you wrote in the table in problem 3b? Explain.

h. If you ignore the context, what is the range for the equation you wrote in the table in problem 3b? Explain.

4. Revisit *Linear Functions and Fitness Plans*.

a. What is the domain of each fitness plan? How do you know?

b. What is the domain of each of the functions you found without context?

c. How are the domains different?

d. How are they related?

5. Revisit *Linear Functions and Fitness Plans*.

a. What is the range of each fitness plan? How do you know?

b. What is the range of each of the functions you found without context?

c. How are the ranges different?

d. How are they related?

6. You’ve studied *y*-intercepts throughout Chapters 2 and 3. In Lesson 3.5, you were asked to find *x*-intercepts without a definition.

a. What is an *x*-intercept?

b. How can you find an *x*-intercept from a graph?

c. How can you find an *x*-intercept in a table?

d. How can you find an *x*-intercept from an equation?

7. Suppose you owe your parents $2000 for a car they bought you. You’re paying it off at a rate of $100 each month.

a. Write an equation to model this situation.

b. What does the slope mean in this situation?

c. What does the *y*-intercept mean in this situation?

d. What does the *x*-intercept mean in this situation?

e. Find the *x*-intercept.

8. Revisit *Systems of Equations in Context, Renting Bicycles*. Solve each problem. Explain!

a. Based on pricing alone, which bike rental company would you not use? Why?

b. Which bike rental company will you use if you want to rent bikes for less than 4 hours?

c. Which bike rental company will you use if you want to rent bikes for more than 4 hours?

d. How do graphs of the equations help you solve problems 8b and 8c?

9. You want to rent a game system and some games to try the system out before you buy it. You check with several companies and find the following information for one-week rentals:

* Game Galaxy charges $50 for the game system rental and $5 per game.
* Electronic Emporium charges $25 for the game system rental and $10 per game.
* Why Buy? doesn’t charge for the game system rental, but charges $15 per game.

a. Complete the table.

|  |  |  |  |
| --- | --- | --- | --- |
| **Rental Company** | **Game System Rental Fee** | **Rental Per Game** | **Equation** |
| **Game Galaxy** |  |  |  |
| **Electronic Emporium** |  |  |  |
| **Why Buy?** |  |  |  |

b. Graph the equations. Estimate where each pair of lines intersects. Write the solutions as ordered pairs.

|  |  |
| --- | --- |
| **Pair of Companies** | **Intersection Point** |
| **Game Galaxy and Electronic Emporium** |  |
| **Game Galaxy and Why Buy?** |  |
| **Electronic Emporium and Why Buy?** |  |

c. Which game system rental company gives the best deal if who want to rent a small number of games (1 or 2 games)? How do you know? Show your work.

d. Which game system rental company gives the best deal for people who want to rent more than 10 games? How do you know? Show your work.

e. Do any of the game system rental companies ever charge the same amount for the same number of games rented? (Note: Include the game system rental fee.)

* If so, which companies and for what number of games?
* How much do they charge for this number of games?

f. Solve the system of equations algebraically for one pair of companies. List the companies. Show your work.

## 3.6 More Systems of Equations

In the 3.5 Homework, you determined how to solve a system of equations graphically. You also determined how to solve a system of equations algebraically when the equations were presented in the form, *y* = *mx* + *b*. In Lesson 3.6, you will play a game to hone your new-found skills then determine and analyze a system for which the equations are not in the form,   
*y* = *mx* + *b*.

### Activity: Linear Sovereignty

Play *Linear Sovereignty* until each player has had 3 turns. Use the gameboard on p. 138.

**Linear Sovereignty Rules.** On your turn:

1. Draw a line that goes through two or more points on the gameboard. **All lines must be functions.** (What types of lines does this rule eliminate?)

2. State a correct equation for the line you drew.

3. Scoring is as follows. You must justify your score.

* + 1 point for each point on the gameboard that is on the line.
  + 1 point for algebraically determining the intersection point of your line with another line. The intersection point cannot be a point already on the gameboard.
  + 1 point for constructing a line parallel or perpendicular to an existing line.

4. If an error is made in finding or justifying an equation or point of intersection, the player correcting the error steals the point(s).

5. Play moves to the left. The game ends when each player has had three turns.

**To Win:** The player with the most points at the end of the game wins.

Answer the following questions when you have finished playing *Linear Sovereignty*:

* How did you determine that two lines were parallel?
* Were you able to determine that two lines were perpendicular? How?
* How can you determine the coordinates of a point of intersection?
* Look at the graph again. How many pairs of parallel lines are possible?
* How many pairs of perpendicular lines are possible?

*Linear Sovereignty* is adapted from *Linear Sovereignty* by Trevor Rice and Charlene Beckmann in Adventures with Mathematics: Climbing from Grade 8 to Algebra 1, Michigan Council of Teachers of Mathematics, 2012, pp. 46–47.



Linear Sovereignty Gameboard

### Activity: Measuring with Pretzels and Cheerios

Use pretzel sticks and cheerios to measure an 8.5-inch by 11-inch sheet of paper. The student page provides an interesting way to measure something if conventional tools are not available. As a bonus, you get to eat the pretzels and the cheerios when you’re done measuring!

Systems of Equations: Pretzels and Cheerios

1. Measure each side of an 8.5-inch by 11-inch sheet of paper using as many pretzel sticks as you can without exceeding the side length of the paper. Fill-in the rest of the length using Cheerios. Record your values in the table below:

|  |  |  |  |
| --- | --- | --- | --- |
| **Side of paper** | **Side length (inches)** | **Number of pretzels, *p*** | **Number of cheerios, *c*** |
| Long side | 11 |  |  |
| Short side | 8.5 |  |  |

2. For each side of the paper, write an equation to show the length in terms of the number of pretzels and the number of cheerios you needed to measure the side length.

3. Graph the equations from problem 2 on the same grid. Let *c* be the independent variable.

4. Do the graphs intersect? If so, what is the point of intersection?

5. What do the coordinates of the intersection point represent in the context?

6. Measure the length of a pretzel stick and the diameter of a Cheerio. Compare your measurements to your solutions in problem 4. Comment on your work based on these comparisons.

*Systems of Equations: Pretzels and Cheerios* is adapted from *Pretzels & Cheerios* by Tiffany Stob, in Adventures with Mathematics: Climbing from Algebra 1 to Geometry, Michigan Council of Teachers of Mathematics, 2010, pp. 19–20.

### 3.6 Homework

1. Solving a system of linear equations in two unknowns is an extension of the equation solving we have been doing all along when the equations are in the form *y* = *mx* + *b.*

a. Solve the system of equations below:

*y* = 4.2*x* – 1.6

*y = 1.7*x *+* 3.8

b. Indicate how the solution process is similar to what you have done in the past when solving equations.

c. Indicate what more you need to do to get a full solution to a system of equations.

2. a. Show how to use an algebraic process to solve the system of equations in *Pretzels and Cheerios.*  Explain why your process works.

b. Is there another algebraic solution process you can use to solve a system of equations? If so, explain the process and use it to solve *Pretzels and Cheerios*.

3. In the cartoon, *Foxtrot*, Paige is frustrated by a system of equations problem she has to solve for homework. (See <https://www.gocomics.com/foxtrot/2009/01/25/>, retrieved April 23, 2020.)

The system is:

2*x* + *y* = 60

*x* + 2*y = 75*

Her older brother asked her, “If 2 shirts and a sweater costs $60 and a shirt and 2 sweaters costs $75, what does each item cost?” Paige solved the problem immediately but didn’t realize she had done so.

a. What was Paige’s answer?

b. Show Paige how to solve the system of equations in her homework. Use two different methods to solve the problem.

## 3.7 Using Quantity Rate Value Tables to Determine Equations

Half the battle in solving a system of equations has to do with finding the equations in the first place. In Lesson 3.7, you will learn a convenient tool to help you set up equations for problems that involve rates.

### Activity: Setting Up Systems of Equations with Quantity Rate Value Tables

This activity guides you in creating a system of equations based on known information in a context. Contexts well-suited for use of QRV tables generally involve two unknown quantities that are added together to get a total, different rates (price per pound, price per ticket, level of acidity, etc.) for each quantity, and a value obtained by multiplying the rate by the corresponding quantity. Read carefully and make sense of each step as you solve the problems below.

1. The admission fee at the Grand Rapids Public Museum is $3 for children and $8 for adults. One day 575 people paid to enter the museum. The museum collected $3450 for admissions.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Quantity** | **Rate** | **Value** |
| **Children** |  |  |  |
| **Adults** |  |  |  |
| **Total** |  |  |  |

a. Fill in the first two rows of the table.

* Initially, the number of Children and the number of Adults are unknown. Choose variables to indicate each of these.
* The rates (ticket prices) are given in the problem.
* How can you use each Quantity and Rate to get the Value?

b. Fill in the empty cells in the last row of the table. The Total Quantity and the Total Value are both given in the problem.

c. Write an equation showing the relationship between the Quantity of Children, the Quantity of Adults, and the Total Quantity.

d. Write an equation showing the relationship between the Value of Children’s tickets, the Value of Adults’ tickets, and the Total Value.

The equations you wrote in problems 1c and 1d provide a system of linear equations.

e. Solve the system in two different ways. Verify that your solution works.

d. How many children and how many adults paid for admission to the museum that day?

2. Melanie’s favorite ways to exercise are playing basketball and running. Her goal is to exercise 60 minutes each day, splitting her time between both basketball and running. She also wants to achieve the equivalent of 10,000 steps per day. Playing basketball for one minute is equivalent to taking 130 steps/minute. Running at the rate of 5 miles per hour is equivalent to taking 185 steps per minute.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Quantity** | **Rate** | **Value** |
| **Playing Basketball** |  |  |  |
| **Running** |  |  |  |
| **Total** |  |  |  |

a. Use the table at right to set up two equations, one showing the relationship between the number of minutes Melanie will play each sport, and one showing the relationship between the equivalent numbers of steps Melanie will take during each activity.

b. Solve the system of equations using two different methods.

c. How many minutes of basketball, *B*, and minutes of running, *R*,should Melanie do each day to exercise 60 minutes and take the equivalent of 10,000 steps?

3. Filiz’s favorite ways to exercise in the summer are walking and cycling. Her goal is to exercise 90 minutes each day, splitting her time between both types of exercise. She also wants to achieve the equivalent of 12,000 steps per day. Walking for one minute at the rate of 3.5 miles per hour is equivalent to taking 130 steps/minute. Cycling at the rate of 15 miles per hour is equivalent to taking 160 steps per minute.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Quantity** | **Rate** | **Value** |
| **Walking** |  |  |  |
| **Cycling** |  |  |  |
| **Total** |  |  |  |

a. Use the table at right to set up two equations, one showing the relationship between the number of minutes Filiz will engage in each type of exercise, and one showing the relationship between the equivalent numbers of steps Filiz will take during each activity.

b. Solve the system of equations using two different methods.

c. How many minutes of walking, *W*, and how many minutes of cycling, *C*,should Filiz do each day to exercise 90 minutes and take the equivalent of 12,000 steps?

What did you learn about using Quantity Rate Value Tables? How can you use what you learned to set-up systems of equations problems?

### 3.7 Homework

1. Use what you learned about Quantity Rate Value Tables (QRV Tables) to solve this problem.

Gummy candy costs $5 per pound. Taffy candy costs $4 per pound. You want to buy 4 pounds of candy and spend exactly $17.50.

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Quantity** | **Rate** | **Value** |
| **Gummy Candy** |  |  |  |
| **Taffy Candy** |  |  |  |
| **Total** |  |  |  |

a. Fill in the first two rows of the table using information from the story.

b. Define your variables. Be specific about what each one means.

c. Write two equations that arise directly from the table.

d. Solve the system of equations. Use two different methods. Show your work. Verify that your solution works.

e. Interpret your solution based on the problem with gummy and taffy candies.

2. Gavin wants to start-up a food truck that uses locally grown ingredients. He plans to sell crepes and smoothies; he will offer different types of each item. In this case, you need a variation of the QRV table to find each equation needed to solve the system.

a. Gavin expects to spend on average $0.70 for ingredients for each crepe and $1.20 for ingredients for each smoothie. He wants to spend only $200 for ingredients each day. Write an equation showing the relationship between costs per item and amount spent on food per day.

b. Gavin wants to sell crepes for $4 each and smoothies for $5 each. He hopes to take in $1000 each day selling just these two items. Write an equation showing the relationship between the prices he will charge and the amount he will take in each day.

c. Use the equations from problems 2a and 2b to determine how many crepes and how many smoothies Gavin should be prepared to sell each day to reach his goals. Solve the problem in at least 2 different ways.

3. QRV tables can also be used to solve mixture problems such as those found in chemistry. Complete the student page, *Chemistry Mixture Problems Using Quantity Rate Value Tables*.

Chemistry Mixture Problems Using Quantity Rate Value Tables

Quantity Rate Value Tables are helpful in setting up equations to solve mixture problems in chemistry and other fields. As you solve these problems, notice that a Rate is needed in all three rows. Add Quantities and Values to obtain equations to solve these problems.

1. How much 10% sulfuric acid (H2SO4) must be mixed with how much 30% sulfuric acid to make 200 milliliters of 15% sulfuric acid?

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Quantity** | **Rate** | **Value** |
| **Weak Acid** |  |  |  |
| **Strong Acid** |  |  |  |
| **Total** |  |  |  |

a. Fill in the first two rows of the table using information from the problem.

b. Define your variables. Be specific about what each one means.

c. Determine equations arising from the table.

d. Solve the system of equations. Show your work. Show that your solution works.

e. Interpret your solution based on the problem with two different strengths of acid solution.

2. Solve these two problems similarly to problem 1. Draw your own table for each problem.

a. You need 20 liters of 20% acid solution. You have jugs of 10% solution and 25% solution. How many liters of each should you combine to get the needed solution?

b. A medical technician has 20% alcohol solution and 70% alcohol solution. She needs 20 liters of a 40% alcohol solution. What amount of each type of solution should she combine?

3. For each problem, add Quantities to get an expression. Add Values to get another expression. Use the rate for the Total row to find a relationship between the expressions.

a. A chemist has 6 liters of a 25% alcohol solution. How much alcohol must he add so that the resulting solution contains 50% alcohol?

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Quantity** | **Rate** | **Value** |
| **Weak Acid** |  |  |  |
| **Strong Acid** |  |  |  |
| **Total** |  |  |  |

b. How many liters of a 14 percent alcohol solution must be mixed with 20 liters of a 50 percent alcohol solution to get a 20 percent alcohol solution? (Draw your own table.)

4. Sterling Silver is 92.5% pure silver. How many grams of pure silver and sterling silver must be mixed to obtain 100g of a 94% Silver alloy?

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Quantity** | **Rate** | **Value** |
| **Sterling Silver** |  |  |  |
| **Pure Silver** |  |  |  |
| **Total** |  |  |  |